Optimal route for drone for monitoring of crop yields

T. Makarovskikh, A. Panyukov, M. Abotaleb, V. Maksimova,
O. Dernova, E. Raschupkin

National Research University SUSU
Chelyabinsk, Russian Federation
[Makarovskikh.t.a,abotalebm,paniukovav]@susu.ru

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Introduction: on Drone and Truck Task

Review

- Makarovskikh, T.; Panyukov, A. Special Type Routing Problems in Plane Graphs. Mathematics 2022, 10, 795. doi: 10.3390/math10050795
- Petunin, A.A.; Polishchuk, E.G.; Ukolov, S.S. On the new Algorithm for Solving Continuous Cutting Problem. IFAC-PapersOnLine 2020, 52, Pp. 2320–2325.
- Petunin, A.A.; Chentsov, A.G.; Chentsov, P.A. Optimal routing in problems of sequential bypass of megacities in the presence of restrictions. Chelyab. Phys.-Math. journal, 2022, 7:2, Pp. 209–233.
- Chung, S.H., Sah, B., Lee, J. Optimization for drone and drone-truck combined operations: A review of the state of the art and future directions. Computers & Operations Research, 2020, 123, ID 105004, doi: 10.1016/j.cor.2020.105004.
- Castro, G.G.R.d. et al. Adaptive Path Planning for Fusing Rapidly Exploring Random Trees and Deep Reinforcement Learning in an Agriculture Dynamic UAVs. Agriculture 2023, 13, 354. doi: 10.3390/agriculture13020354
- Tian H. et al. (2023) Design and validation of a multi-objective waypoint planning algorithm for UAV spraying in orchards based on improved ant colony algorithm. Front. Plant Sci. 14:1101828. doi: 10.3389/fpls.2023.1101828
- Lobaty A.A., Bumai A.Y., Avsievich A.M. Formation of unmanned aircraft trajectory when flying around prohibited areas. System analysis and applied information science. 2021, 4, Pp. 47–53. (In Russ.) https://doi.org/10.21122/2309-4923-2021-4-47-53
- Rudenko E.M., Semikina E.V. Intelligent monitoring of UAV group on eyler-hamilton reference graphs on the local. Institute of Engineering Physics. 2021. Vol. 62. No. 4. Pp.

Statement of the Problem and Constraints (1)

- a farm that owns N fields,
- each field is inscribed in a rectangle $F_i = \{l_i \times w_i\}$, $i = \overline{1, N}$,
- the coordinates of the upper left corner of each such rectangle are saved in WGS84[a]
- dimensions of the rectangle are l_i and w_i, $i = \overline{1, N}$.

[a] World Geodetic Parameters of the Earth 1984, which includes the system of geocentric coordinates. Unlike local systems, it is a single system for the entire planet.

Constraints

- C_{call}: one rising of a drone cost
  - the working time of an operator and one battery charge,
  - the number of charges for one battery is limited,
  - the cost of a new battery is proportionally divided between all its charges
- C_{way}: the cost of transfer the brigade truck to the place of shooting;
- characteristics of the selected drone for research:
  - T_{av}: average flight time before recharging,
  - average velocities of ascent to a given height V_{up} and flight V_{flight};
- S_a: the size of an area taken from above;
- a_x, a_y: camera aspect ratio;
- h: the height of monitoring;
- $C_{recharge}$: drone restart cost;
- $P \in [0; 1)$: image overlay percentage;
- drone returns for recharging to the truck that is placed in a starting point or moves to any allowed point at the boundary of a field, and continues monitoring after recharging.

Task

- to search a drone flight path for the given map of areas that satisfies the imposed constraints and parameters,
- determine the optimal starting point,
- minimize the number of drone recharges and minimize the distances between the launch points of the device

Stages

- Let flight time is not limited and explored area is connected
- Let the flight time is not limited but explored area is not connected.
- Let the flight time is limited by the drone battery capacity, but the location of start is any point. The field is not connected.
- The general case when the flight time is limited by battery capacity, and location of possible starting points is fixed due to some restrictions
- The general case as the previous one, but the investigated area has any shape, not only rectangle.
- To take into account the influence the external factors to improve the drone flight trajectory.

Known Models

[Chung2020]:

- several mathematical models and problems based on the concept of DTCO (TSPD+VRPD)
- the delivery task when the aim is to determine the order of drone flyby or the territory surved (FSTSP, PDSTSP [Murray2015], PDSTSP+DP [Ham2018], TSP-D [Agatz2018] etc.)

[Chung2020] Chung, S.H., Sah, B., Lee, J. Optimization for drone and drone-truck combined operations: A review of the state of the art and future directions. Computers & Operations Research, 2020, 123, ID 105004, doi: 10.1016/j.cor.2020.105004.
[Murray2015] Murray, C.C., Chu, A.G., 2015. The flying sidekick traveling salesman problem: optimization of drone-assisted parcel delivery. Transp. Res. C Emerg. Technol. 54, 86вЂ“109.
[Ham2018] Ham, A.M. Integrated scheduling of m-truck, m-drone, and m-depot constrained by time-window, drop-pickup, and m-visit using constraint programming, Transportation Research Part C. Emerging Technologies, 2018, 91, Pp. 1–14, doi: 10.1016/j.trc.2018.03.025.
[Agatz2018] Agatz, N., Bouman, P., Schmidt, M., 2018. Optimization approaches for the traveling salesman problem with drone. Transp. Sci. 52 (4), Pp. 965–981.

Considered Task

- we discuss the drone trajectory used for monitoring of crop yields, i.e. the minimal sequence of rectangles covering the investigated area
- we need one drone and one truck
- the nodes are divided into three categories:
  - drone node: a node visited by a drone only,
  - truck node: a node visited by a truck only,
  - combined node: a node visited by a truck and a drone.
- Approaches: [Agatz2018], [Makarovskikh2022]
- Solution: the combination of a truck route (composed of the truck and combined nodes) and a drone route (composed of the drone and combined nodes)
- the considered case of yields monitoring is to be a particular case allowing solving our task by polynomial time

[Agatz2018] Agatz, N., Bouman, P., Schmidt, M., 2018. Optimization approaches for the traveling salesman problem with drone. Transp. Sci. 52 (4), Pp. 965–981.
[Makarovskikh2022] Makarovskikh, T.; Panyukov, A. Special Type Routing Problems in Plane Graphs. Mathematics 2022, 10, 795. doi: 10.3390/math10050795

Graph Model

- Plane graph $G_{F_i} = (V_{F_i}, E_{F_i})$ for each investigated area (field) F_i,
- the set of vertices V_{F_i} of which be the points of shooting by the drone camera,
- E_{F_i} be the connections between the nearest neighbours.

Each $S_a = (a_x \cdot K) \cdot (a_y \cdot K) = K^2 \cdot a_x a_y$, then lengths of each photo size are equal to

$$X = a_x \cdot K = a_x \cdot \sqrt{\frac{S_a}{a_x a_y}}, \quad Y = a_y \cdot K = a_y \cdot \sqrt{\frac{S_a}{a_x a_y}}.$$

The point of shooting be the center of the picture.
The point of the next shooting for current shot (x_{cur}, y_{cur}) with overlay P:

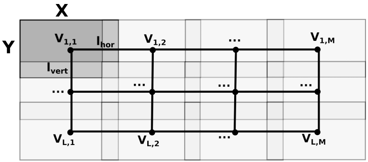
$$x_{next} = x_{cur} + X - X \cdot P, \quad y_{next} = y_{cur}$$

for moving along the rectangle width or

$$x_{next} = x_{cur}, \quad y_{next} = y_{cur} + Y - Y \cdot P$$

for moving along the rectangle height.

Graph Model (2)



- The optimal path $P(F_i)$ for the rectangular area F_i
- we need moving only horizontally or vertically,
- Vertices: $v_{ij} \in V_{F_i}$, $i = \overline{1, M}$, $j = \overline{1, L}$, $|V_{F_i}| = M \cdot L$,
- Edges $e \in E_{F_i}$ correspond to possible moves from each vertex, $|E_{F_i}| = (M-1) \cdot (L-1)$.

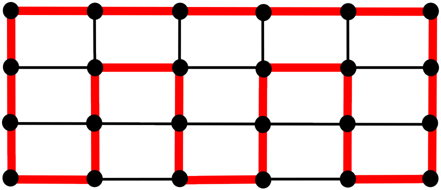
Features

- One half of edges $e \in E_{F_i}$ has length $l_{vert} = Y - Y \cdot P$
- another one has length $l_{hor} = X - X \cdot P$.
- Task: to define the Hamiltonian cycle of the shortest length in this graph.
- Since $a_x \neq a_y$, without loss of generality, let's consider that $a_x > a_y$, and $l_{vert} < l_{hor}$.
- half of edges is shorter then the others, then to get the shortest path we need:
  - include as much as possible these short edges of length l_{vert};
  - as less as possible of long edges of length l_{hor}.
- in any case the minimal number of long edges in a route is equal to $2 \cdot (M - 1)$.

Cases for the shortest path of flyby a single F_i

Case 1: M is even for any L. The length of the route is equal to

$$R = 2(M-1)l_{hor} + (2(L-1) + (M-2)(L-2))l_{vert};$$



Cases for the shortest path of flyby a single F_i

Case 2: M is odd, L is even. The length of the route is equal to:

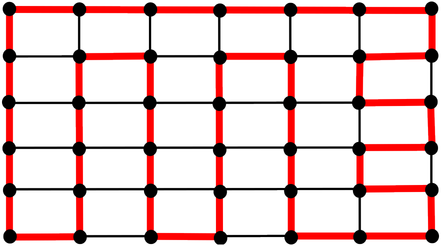
a. Avoiding idle passes:

$$R = (2(M-1) + (L-2))l_{hor} + (2(L-1) + (M-2)(L-2))l_{vert}$$

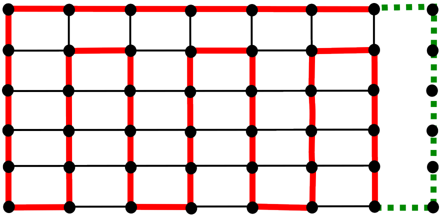
b. Allowing idle passes:

$$R = 2(M-1)l_{hor} + ((L-1) + (M-1)(L-2))l_{vert} + L_{idle},$$

$$L_{idle} = 2l_{hor} + (L-1)l_{vert}$$



(a)



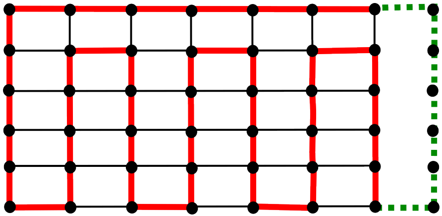
(b)

Cases for the shortest path of flyby a single F_i

Case 3: both M and L are even. The length of the route is calculated the same way as for the previous case with idle pass.

$$R = 2(M-1)l_{hor} + ((L-1) + (M-1)(L-2))l_{vert} + L_{idle},$$

$$L_{idle} = 2l_{hor} + (L-1)l_{vert}$$



Algorithm for Shortest Connecting of Areas

- Let the farmer needs monitoring of more than one field.
- If the boundaries of the fields are combined or are at a distance of several tens of meters these fields can be united to one investigated area.
- In common fields may lie in distance of several kilometres from each other.
- The approach used for connecting the different areas under research may be similar to [Petunin2022].

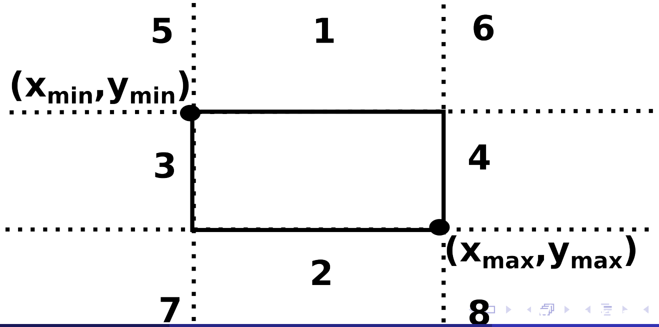
[Petunin2022] Petunin, A.A.; Chentsov, A.G.; Chentsov, P.A. Optimal routing in problems of sequential bypass of megacities in the presence of restrictions. Chelyab. Phys.-Math. journal, 2022, 7:2, Pp. 209–233.

Peculiarities of the applied task

Constraint: if the distance between boundaries of two fields takes time less than drone landing and takeoff time then these fields are connected to one object.

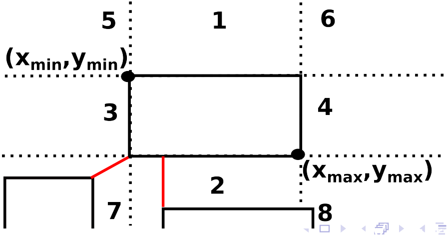
Task: Define the shortest distance between boundaries of fields F_i. We use only vertices of graph G belonging to outer face.

All fields F_i are considered to be rectangles. Consider the rectangle $F_i = \{(x^i_{min}, y^i_{min}), (x^i_{max}, y^i_{max})\}$. All the outer space of this rectangle is divided into 8 areas.

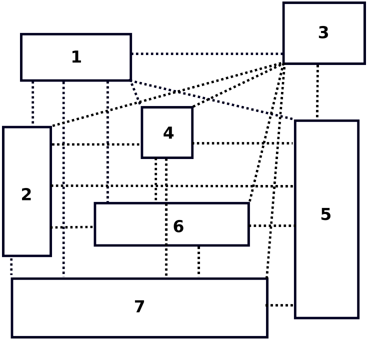


The nearest rectangles (List of Rules)

1. If the straight line l passing along the boundary of rectangle F_i, $i = \overline{1, N}$ intersects the boundary of rectangle F_j, $j \neq i$, $j = \overline{1, N}$ then the segment of l between F_i and F_j be the shortest way between them.
2. For each pair of rectangles F_i and F_j, $j \neq i$, $i, j = \overline{1, N}$ there exists the segment connecting them.
3. If there are 2 segments A and B between rectangles F_i and F_j, $j \neq i$, $i, j = \overline{1, N}$, and their lengths $L(A) = L(B)$ then for any segment C between A and B the equality $L(A) = L(B) = L(C)$ holds.
4. If F_j belongs to areas 5–8 of rectangle F_i outer space, $j \neq i$, $i, j = \overline{1, N}$, then the shortest segment connects one of the following pairs of points:
    - (x^i_{min}, y^i_{min}) and (x^j_{max}, y^j_{max}) (area 5 of F_i) or (x^j_{min}, y^j_{min}) and (x^i_{max}, y^i_{max}) (area 8 of F_i);
    - (x^i_{min}, y^i_{max}) and (x^j_{max}, y^j_{min}) (area 6 of F_i) or (x^j_{max}, y^j_{min}) and (x^i_{min}, y^i_{max}) (area 7 of F_i).

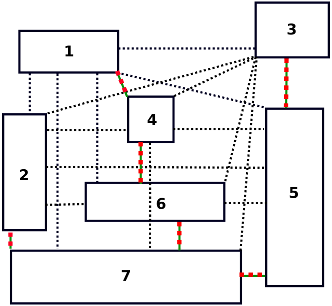


The shortest connections between rectangles belonging to different areas



- Using these rules we can calculate the shortest connections between all pairs of rectangles F_i, $i = \overline{1, N}$.
- If F_i be the vertices of the connections graph G_c:
  - its edges $e \in E(G_c)$ be the connections,
  - weight $w(e)$ of each $e \in E(G_c)$ be the length of the segment.
- $G_c = K_{|F_i|}$.

The minimal spanning tree for the given flight plan



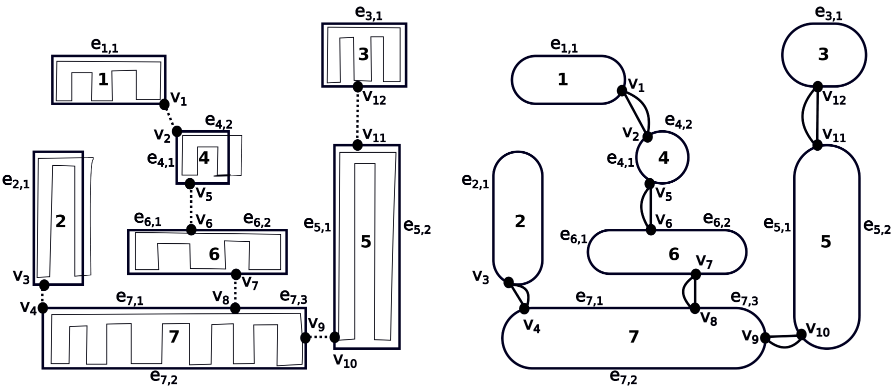
- To get the shortest connection between F_i we need define the minimal spanning tree $T(G_c)$

Theorem 1

The obtaining the minimal length connections between all F_i, $i = \overline{1, N}$ can be run in polynomial time.

Flyby graph

We have connected graph $G_p = (V_p, E_p)$, $E_p = E(P(F_i)) \cup E(T(G_c))$ corresponding to a drone path.



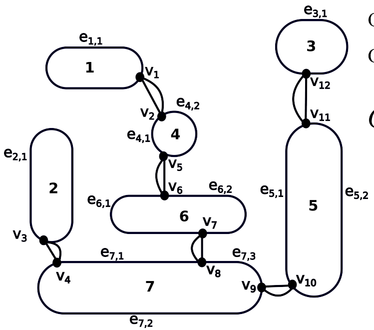
(a) (b)

Flyby Graph (Explanation)

a in common the degree of $v_i \in V_p$ may be either odd or even. So, to fly by the defined path with the defined connections and return to initial point (drone returns to truck) we need pass edges $E(T(G_c))$ connecting the odd vertices twice or define the shortest matching between them.

b the odd vertices are connected twice, we have connected 4-regular graph with cut vertices

AOE-chain for plane connected 4-regular graph



one of the possible solutions of the considered graph:

$$C(G_p) = v_1 e_{1,1} v_2 e_{4,1} v_5 v_6 e_{6,1} v_7 v_8 e_{7,1} v_4 v_3 \\ e_{2,1} v_3 v_4 e_{7,2} v_9 v_{10} e_{5,2} v_{11} v_{12} e_{3,1} \\ v_{12} v_{11} e_{5,1} v_{10} v_9 e_{7,3} v_8 v_7 e_{6,2} v_6 v_5 \\ e_{4,2} v_2 v_1.$$

Drone path for the limited time of flight

1. drone node: a node visited by a drone only, they are the points of a field F_i not belonging boundary or nodes marked as unreachable by truck;
2. combined node: a node visited both by a truck and a drone (for both monitoring and recharging), it is a node belonging boundary of the field reachable by truck.

Rules of Recharging

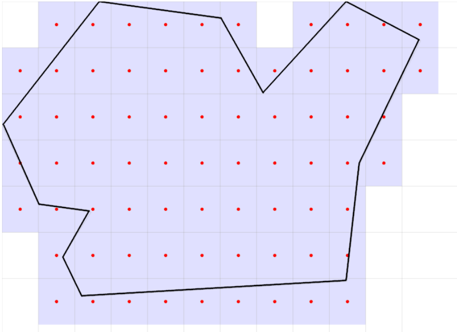
1. $G_p = (V_p, E_p)$, $V_p = V_{drone} \cup V_{truck}$, $V_{comb} = V_{drone} \cap V_{truck}$.
2. Recharging points: $T_{flight} = T_{up} + T_{explore} + T_{down}$, where
   - T_{up} be the time for rising the drone,
   - T_{down} be the time for drone descent,
   - $T_{explore}$ be the time for shooting.
3. Ideal case: $S_{flight} = V_{flight} \cdot T_{explore}$ be the maximal length of drone path.
4. For safe drone descent we need pass the way shorter than S_{flight}
   - drone goes down in the nearest combined node $v^* \in V_{comb}$ before length S_{flight} is reached
   - we put the truck to node v^*.
   - Knowing the way for a drone $C(G_p)$ we can define the nodes $v_1^*, v_2^*, \ldots v_k^*$ where the truck waits for drone for recharging.

Drone Optimal Path with Recharges

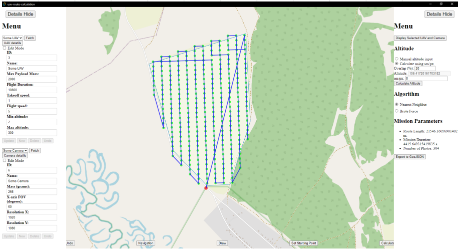
- Use algorithm PPOE-Cover [Makarovskikh2022] where all the vertices of graph (nodes) are defined as:
  - V_{in} (be the starting points of chains)
  - V_{out} (be the ending nodes of chains)
- Algorithm PPOE-Cover defines the path in a graph with optimal length connections between odd vertices.

Open tasks: take outer factors into account

Computational experiment



- Discretisation algorithm: the rectangles included in the survey area are highlighted in purple, the survey points are marked in red.



- Application for obtaining the drone flight path.

Conclusion

- The considered type of drone and truck problem has lot of unsolved tasks concerning different technological constraints.
- Restrictions on flight time and height, on placement of truck, on properties of camera, and weather conditions are among them.
- We considered the problem of planning the drone path with several easy constraints: we run our drone and truck system in the ideal weather (no wind, positive temperature), and suppose that truck can parking in almost all outer border points.
- We defined the way of obtaining the drone flight trajectory for monitoring the objects of interest to the customer and determining the number and place for recharging. For this purpose we used the developed earlier polynomial routing algorithm.