CHARACTER MATRICIES OF PRIMARY CYCLIC GROUPS

R. ZH. ALEEV, V. N. ALEEVA

In the work of Higman[1], an approach was developed to study the units of the integer group rings of finite Abelian groups using characters. The foundation for this method is the description of the relationship between two bases: the group basis and the basis of minimal idempotents.

In [2], the units of the integer group rings of cyclic groups of prime orders were studied. The next step is to study the units of the integral group rings of cyclic groups of orders p^{n+k}, where p is a prime integer and $n+k \geq 2$. In this case, the relationship between these bases will be quite complicated. We simplify this relation for groups of order p^{n+k} based on the extraction of the character matrix of a subgroup of order p^n, where $n, k \in \{1, 2, \dots\}$. Unfortunately, the results obtained are very cumbersome, so we will limit ourselves to the case when $k = 1$.

Further, we use the following notations.

(1) p is a prime integer.
(2) $G = \langle x \rangle$ is a cyclic group of order p^{n+1} and $P = \langle x^p \rangle$ is its subgroup of order p^n.
(3) α is a primitive p^{n+1}-th root of unity.
(4) For any $j, k \in \{0, 1, \dots, p^{n+1} - 1\}$ we have the character $\chi_j(x^k) = \alpha^{jk}$.
(5) $e(\chi)$ is the minimal idempotent determined by the character χ of the group G.

An arbitrary element u of the complex group algebra $\mathbf{C}G$ when decomposed by the group basis and the basis of minimal idempotents has the form:

$$u = \sum_{s=0}^{p-1} x^s \sum_{j=0}^{p^n-1} \gamma[s]_j x^{pj} = \sum_{s=0}^{p^n-1} \sum_{k=0}^{p-1} \beta[s]_k e(\chi_{sp+k}).$$

For any $j \in \{0, 1, \dots, p-1\}$ we put

$$\vec{\Gamma}[j] = (\gamma[j]_0, \gamma[j]_1, \dots, \gamma[j]_{p^n-1}) \text{ и } \vec{B}[j] = (\beta[j]_0, \beta[j]_1, \dots, \beta[j]_{p^n-1}).$$

Theorem. *With the introduced notations, the following conditions are equivalent.*

(1) *The element u belongs to $\mathbf{C}P$.*
(2) *$\vec{\Gamma}[j] = \vec{0}$ for any $j \in \{1, 2, \dots, p-1\}$.*
(3) *$\vec{B}[j] = \vec{B}[0]$ for any $j \in \{1, 2, \dots, p-1\}$.*

Список литературы

[1] G. Higman, The units of group rings. *Proc. London Math. Soc(2).*, **46**: 1 (1940), 231–248.
[2] R. Zh. Aleev, Krugovye edinicy v gruppovyh kolcah konechnyh abelevyh grupp. *Algebra i logika: Mater. mezhd. ross-kit. seminara.* Irkutsk: Izd. Irkut. gos. ped. un-ta, 2007, 11–14.(in Russian)

South Ural State University, Chelyabinsk State University, Chelyabinsk (Russia)
Email address: aleevrz@susu.ru, aleevavn@susu.ru