UNIT GROUPS OF THE INTEGRAL GROUP RINGS OF CYCLIC GROUPS OF ORDERS $2p$, WHERE $p \geq 5$ IS A PRIME

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In [1], the units of the integer group rings of cyclic groups of prime orders were studied. Here we consider the units of the integral group rings of cyclic groups of orders $2p$ for a prime $p \geq 5$.

Further, we use the following notations.

(1) $p \geq 5$ is a prime integer.
(2) $G = \langle x \rangle$ is a cyclic group of order $2p$.
(3) α is a primitive p-th root of unity.
(4) χ is the character of the group G for $\chi(x) = \alpha$.
(5) $\mathbf{Q}(\chi)$ is the character field of χ.

According to [2], for the character χ and the element $\mu \in \mathbf{Q}(\chi)$, we define an element of the rational group algebra $\mathbf{Q}G$,

$$u_\chi(\mu) = 1 + \sum_{\varphi \in \mathrm{Aut}(\mathbf{Q}(\chi))} (\varphi(\mu) - 1)\, e(\varphi(\chi)),$$

where $e(\varphi(\chi))$ is the minimum idempotent corresponding to the character $\varphi(\chi)$.

Let g be a primitive root modulo p. Denote by

$$\mu_0 = \frac{1 - \alpha^g}{1 - \alpha} = 1 + \alpha + \cdots + \alpha^{g-1}.$$

Let f be the multiplicative order of 2 modulo p and

$$r = \mathrm{Lcm}\left(\frac{p-1}{2}, 2^f - 1\right).$$

Theorem. *Let* $\mathrm{Un}(\mathbf{Z}\langle x^2 \rangle)$ *be a unit group of the integral group ring of a subgroup of order* p. *For* $m \in \{1, 2, \ldots, p-1\}$, *we denote by* φ_m *an automorphism of the field* $\mathbf{Q}(\chi)$ *such that* $\varphi_m(\alpha) = \alpha^m$. *Then*

$$\mathrm{Un}(\mathbf{Z}\langle x^2 \rangle) \times \prod_{k=0}^{(p-5)/2} \langle u_\chi(\varphi_{g^k}(-\mu_0^r))$$

has a finite index in $\mathrm{Un}(\mathbf{Z}G)$.

References

[1] R. Zh. Aleev, Krugovye edinicy v gruppovyh kolcah konechnyh abelevyh grupp. *Algebra i logika: Mater. mezhd. ross-kit. seminara.* Irkutsk: Izd. Irkut. gos. ped. un-ta, 2007, 11–14.(in Russian)

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